C2 Exercise 2D (cosine rule)

Note Title 24/01/2007

In this exercise use the cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

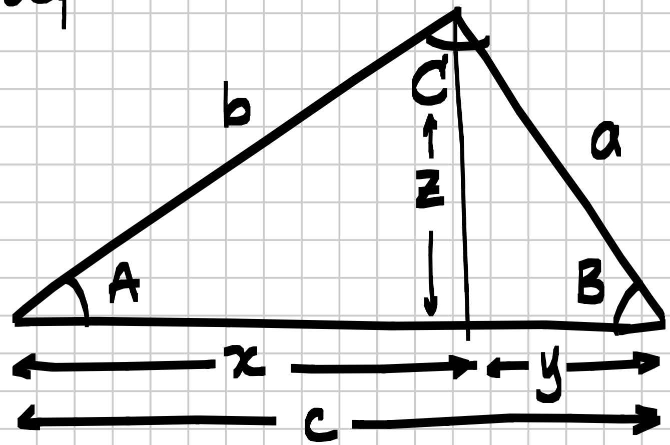
or its rearrangement:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

You may use the cosine rule to find an unknown side in a triangle when you know the lengths of two sides and the size of the angle between them.

You can, of course, permute the letters a, b, c and A, B, C respectively if this is helpful.

Proof 1:



$$b^2 = x^2 + z^2 \quad \text{by Pythagoras' Thm.}$$

$$y = c - x$$

$$\text{so } a^2 = z^2 + (c-x)^2$$

$$b^2 - a^2 = x^2 \cancel{+z^2} \cancel{-z^2} - (c-x)^2$$

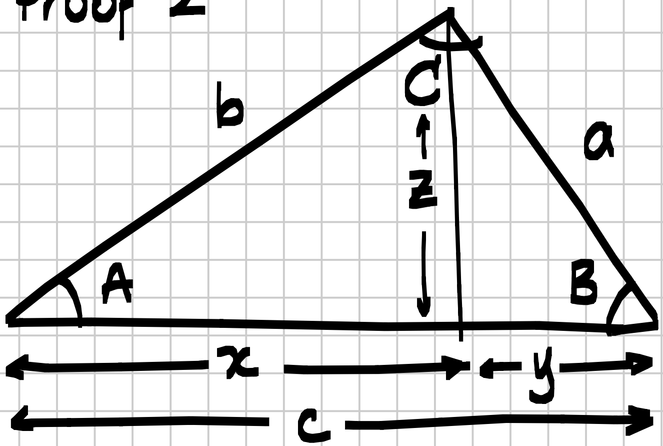
$$\Rightarrow \quad b^2 - a^2 = x^2 - (c^2 - 2cx + x^2)$$

$$\Rightarrow \quad b^2 - a^2 = -c^2 + 2cx$$

$$\text{but} \quad x = b \cos A \quad \text{so} \quad b^2 = a^2 - c^2 + 2bc\cos A$$

$$\Rightarrow \quad a^2 = b^2 + c^2 - 2bc\cos A \quad \square$$

Proof 2



① $\cos A = \dfrac{x}{b}$

but $x^2 = b^2 - z^2$

and $z^2 = a^2 - y^2$

with $y = c - x$, $z^2 = a^2 - (c-x)^2$

So $x^2 = b^2 - \left[a^2 - (c-x)^2\right] \Rightarrow x^2 = b^2 - a^2 + c^2 - 2cx + x^2$

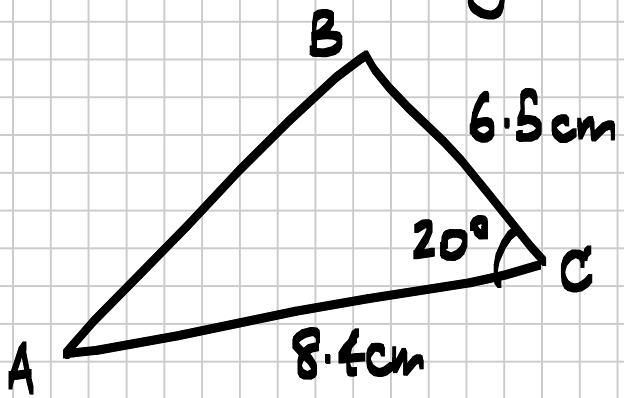
$\Rightarrow 2cx = b^2 + c^2 - a^2 \Rightarrow$ ② $x = \dfrac{b^2 + c^2 - a^2}{2c}$

substituting ② into ①: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ $\square$.

(this is the 'rearranged' version)

C2 Ex 2D

1 Calculate the length of the third side:



$$c^2 = a^2 + b^2 - 2ab\cos C$$

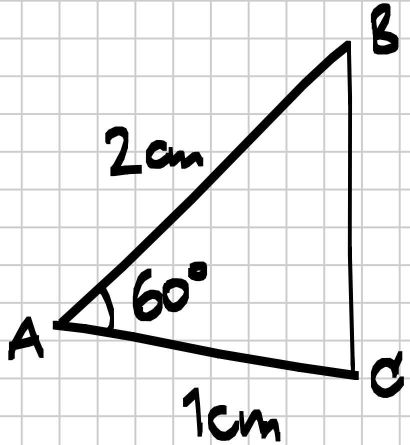
$$c^2 = 10.19556581$$

$$c = 3.19304961 \approx 3.19\text{cm}$$

C2 Ex 2D

1 Calculate the length of the third side:

b)



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 1^2 + 2^2 - 2\times 1\times 2\cos 60°$$

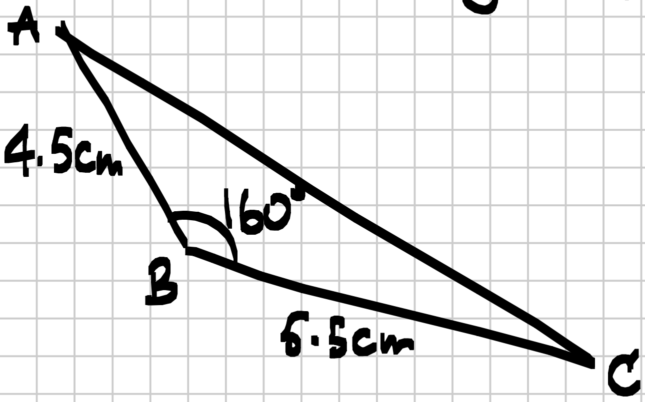
$$a^2 = 3$$

$$a = \sqrt{3} = 1.732050808 \simeq 1.73\text{ cm}$$

C2 Ex 2D

1 Calculate the length of the third side:

c



$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = 5.5^2 + 4.5^2 - 2\times 5.5\times 4.5\times \cos 160^\circ$$

$$b^2 = 97.01478473$$

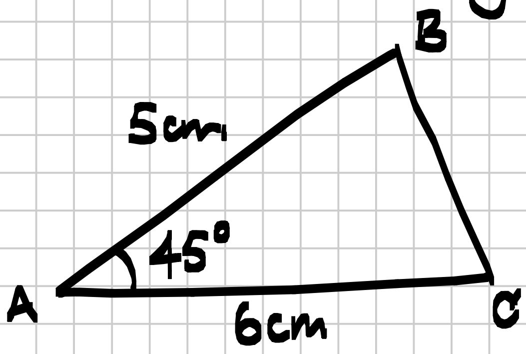
$$b = 9.849608354$$

$$b \approx 9.85\ \text{cm}$$

C2 Ex 2D

1 Calculate the length of the third side:

d



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 6^2 + 5^2 - 2\times5\times6\cos 45$

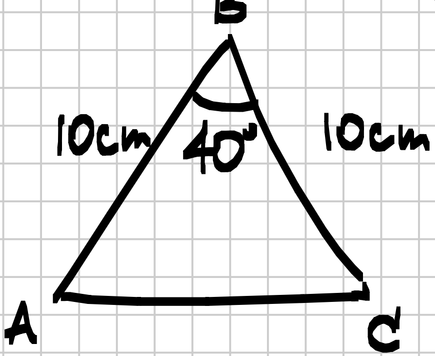
$a^2 = 18.57359313$

$a = 4.30970917 \approx 4.31\text{ cm}$

C2 Ex 2D

1 Calculate the length of the third side:

e



$b^2 = a^2 + c^2 - 2ac\cos B$

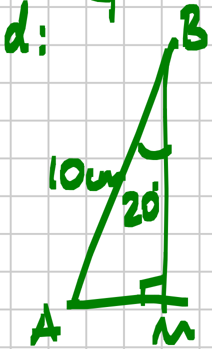
$b^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \cos 40$

$b^2 = 200 - 200\cos 40 = 46.79111138$

$b = 6.840402867$

$b \simeq 6.84$ cm

You might have split this isosceles triangle into two right-angled triangles instead:



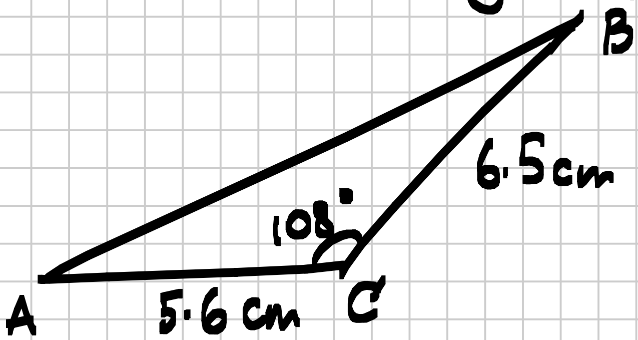
$AM = 10\sin 20 = 3.420201433$

$\Rightarrow AC \simeq 6.84$ cm

C2 Ex 2D

1 Calculate the length of the third side:

f



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 6.5^2 + 5.6^2 - 2 \times 6.5 \times 5.6 \cos 108$

$c^2 = 96.10643719$

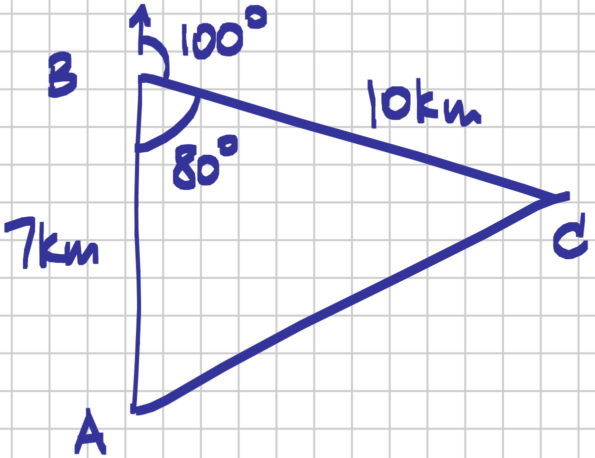
$c = 9.803389067$

$c \simeq 9.80\text{ cm}$

⚠ rules on 3 s.f. zeros count once you've started counting.

C2 Ex 2D

2 From a point A a boat sails due north 7km to B. The boat leaves B and moves on a bearing of 100° for 10km until it reaches C. Calculate the distance of C from A.



distance of C from A is b

$b^2 = a^2 + c^2 - 2ac\cos B$

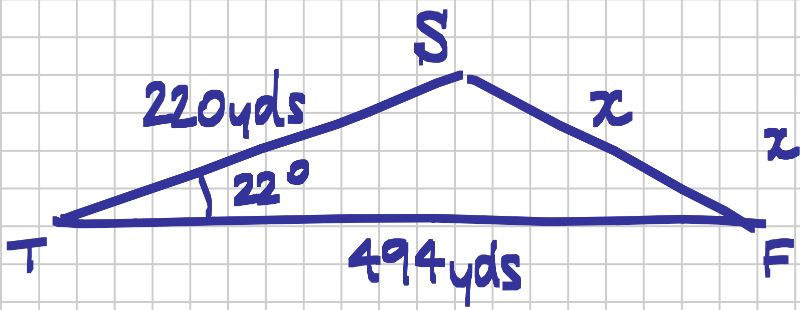
$b^2 = 10^2 + 7^2 - 2\times10\times7\cos 80°$

$b^2 = 124.6892554$

$b = 11.16643431 \simeq 11.2\text{km}$

C2 Ex2D

3 The distance from the tee, T, to the flag, F, on a particular hole on a golf course is 494 yards. A golfer's tee shot travels 220 yards and lands at a point S, where $\angle STF = 22°$. Calculate how far the ball is from the flag.



Call remaining distance x

$$x^2 = 494^2 + 220^2 - 2 \times 494 \times 220 \cos 22$$

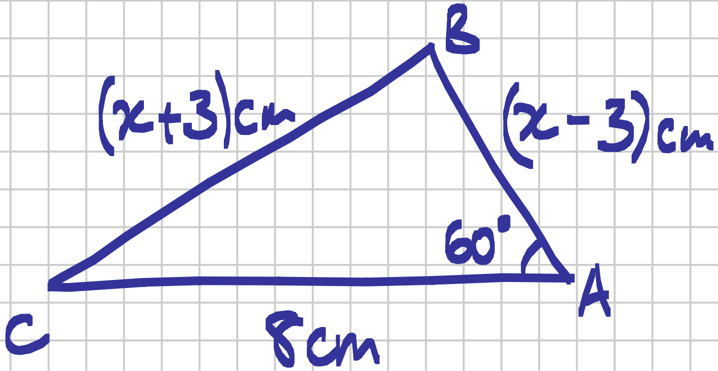
$$x^2 = 90903.31737$$

$$x = 301.5017701...$$

$$x \approx 302 \text{ yds.}$$

C2 Ex 2D

4 In $\triangle ABC$, $AB = (x-3)$ cm, $BC = (x+3)$ cm, $AC = 8$cm, and $\angle BAC = 60°$.
Use the cosine rule to find the value of x.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$(x+3)^2 = 8^2 + (x-3)^2 - \cancel{2}\times 8(x-3)\times\frac{1}{\cancel{2}}$$

$$\Rightarrow \cancel{x^2} + 6x + \cancel{9} = 64 + \cancel{x^2} - 6x + \cancel{9} - 8x + 24$$

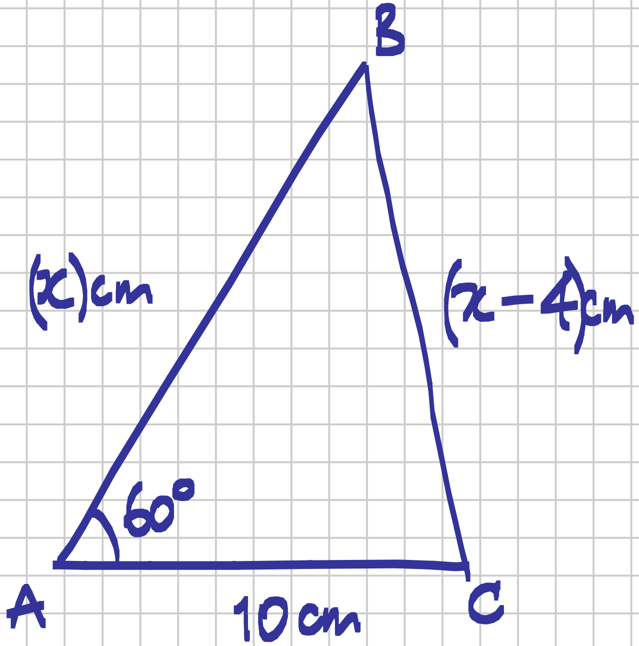
$$\Rightarrow 20x = 88$$

$$\Rightarrow x = 4.4$$

C2 Ex 2D

5 In $\triangle$ ABC, AB = x cm, BC = $(x-4)$ cm, AC = 10cm and $\angle BAC = 60°$. Find the value of x.

Have you encountered a similar problem before?



$$(x-4)^2 = x^2 + 10^2 - \cancel{2} \times 10x \cancel{\cos 60°}$$

$$\cancel{x^2} - 8x + 16 = \cancel{x^2} + 100 - 10x$$

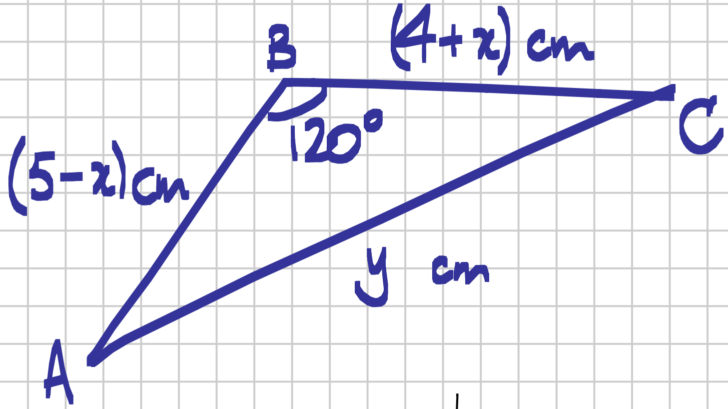
$$2x = 84$$

$$x = 42$$

C2 Ex2D

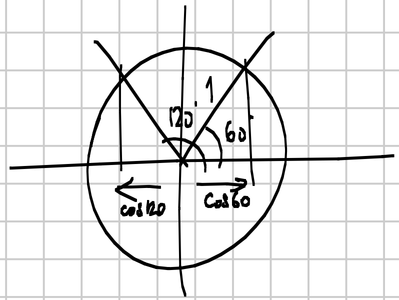
6 In $\triangle ABC$, $AB = (5-x)$ cm, $BC = (4+x)$ cm, $\angle ABC = 120°$ and $AC = y$ cm

a Show that $y^2 = x^2 - x + 61$



$$y^2 = (5-x)^2 + (4+x)^2 + \cancel{2}(5-x)(4+x)\cos \cancel{120°}$$

$\cos 120° = -\cos 60 = -\frac{1}{2}$ * so $-2\cos 120 = 1$



$$y^2 = 25 - 10x + x^2 + 16 + 8x + \cancel{x^2} + 20 + x - \cancel{x^2}$$

$$y^2 = x^2 - x + 61 \;. \square$$

6b Use the method of completing the square to find the minimum value of y^2 and give the value of x for which it occurs.

$$y^2 = x^2 - x + 61$$

$$\Rightarrow \quad y^2 = \left(x - \tfrac{1}{2}\right)^2 - \tfrac{1}{4} + 61$$

$$y^2 = \left(x - \tfrac{1}{2}\right)^2 + 60\tfrac{3}{4}$$

so minimum value of y^2 is $60\tfrac{3}{4}$.

this occurs at $x = \tfrac{1}{2}$.